

DOCUMENT RESUME

ED 443 726

SE 063 957

AUTHOR Pligge, Margaret A.; Kent, Laura B.; Spence, Mary S.
TITLE Examining Teacher Change within the Context of Mathematics Curriculum Reform: Views from Middle School Teachers.
PUB DATE 2000-04-00
NOTE 32p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 24-28, 2000). Some figures may not reproduce well.
PUB TYPE Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Attitudes; *Educational Change; *Mathematics Curriculum; Mathematics Education; Mathematics Teachers; Middle School Teachers; Middle Schools; *Pedagogical Content Knowledge; Professional Development; *Teacher Improvement

ABSTRACT

This article describes teacher change using the backdrop of a standards-based reform mathematics curriculum for middle grades and direct quotes from teachers and math support coordinators involved in the implementation over the last five years. Each of the sixteen participants had at least one year of experience teaching or supervising the instruction of the curriculum. The curriculum, "Mathematics in Context: A Connected Curriculum for Grades 5-8," was designed to build instruction on students' informal knowledge using meaningful context situations. The results of survey, interview, and classroom observation data illustrate that, for these selected teachers and support staff, both the design of the curriculum and the staff development workshops changed their perceptions of both what mathematics is as a subject and how mathematics should be taught. The quotes from the participants and the specific examples that they refer to in the curriculum provide evidence of the legitimacy of their perceptions about how they have changed their beliefs about teaching and learning as a result of their interactions with this particular curriculum. The paper concludes with a discussion of the impact of the reform in general and the continued vision shared by mathematics educators to help all students learn significant mathematics. (Contains 31 references.) (YDS)

U.S. DEPARTMENT OF EDUCATION
OFFICE OF EDUCATIONAL RESEARCH AND IMPROVEMENT
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

M. Pligge

U.S. DEPARTMENT OF EDUCATION
OFFICE OF EDUCATIONAL RESEARCH AND IMPROVEMENT
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

1

U.S. DEPARTMENT OF EDUCATION
OFFICE OF EDUCATIONAL RESEARCH AND IMPROVEMENT
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

☒ This document has been reproduced as
received from the person or organization
originating it.

☐ Most changes have been made to
improve readability quality.

☐ Points of view or opinions stated in this
document do not necessarily represent
official OERI position or policy.

Examining Teacher Change Within the Context of Mathematics Curriculum Reform: Views from Middle School Teachers

Margaret A. Pligge – University of Wisconsin Madison

Laura B. Kent – University of South Carolina

Mary S. Spence – American Institute for Research

Paper presented at the annual meeting of the
American Educational Research Association
New Orleans, LA
April, 2000

BEST COPY AVAILABLE

Examining Teacher Change Within the Context of Mathematics Curriculum Reform: Views from Middle School Teachers

One thing is how connected everything is. I learned math in a very fragmented way and I think that influenced how I taught. I now see a powerful organizing principle that underlies mathematics and I think that makes a big difference in my teaching.
(6th Grade Mathematics Teacher)

We have learned many valuable lessons from the reform movement of the 90's in education. One of those lessons is that forcing change on those unwilling or unable to change their practice can be disastrous for both students and teachers alike. Many negative stories of rebellion and going "back to basics" receive widespread attention, both within education and the wider community. Yet, there are also lessons to be learned from the flip side of this negativity--namely, that for many teachers, implementing a new curriculum provides opportunities for them to learn new content and improve their instruction.

This article explores teachers' perceptions of how their content knowledge and their teaching strategies changed as a result of implementing a reform middle school curriculum called *Mathematics in Context: A Connected Curriculum for Grades 5-8* or MiC (NCRMSE & Freudenthal Institute, 1997-98). Fourteen middle school teachers and two math support professionals from eight states volunteered to participate in the study. Each teacher had at least one year teaching MiC and had participated in one or more professional development workshops. Some of the participants evolved into consultants for implementing the program, participating as staff development leaders for new MiC implementation sites. These teachers were selected because one or more of the

authors noted a positive growth in their beliefs, math content and pedagogical content knowledge and classroom assessment practices. We do not claim that the curriculum alone was the impetus for change. It would be much too simplistic to say that any one curriculum has the power to change teacher's beliefs about how and what mathematics should be taught. Nor do we claim that all teachers implementing MiC change in the same ways these teachers changed. Indeed, there are some teachers who use MiC and do not change their practice and/or do not fully participate in successfully teaching the curriculum. A discussion of the full implementation of MiC is beyond the scope of this article.

We focus instead on teachers and leaders who embraced change at the school and classroom level. Their self-report data, like the quote at the beginning of this article, tells a story of powerful reflections that provide evidence of real change in terms of both teaching practices and beliefs about how children learn mathematics. In using their self-report data, we attempt to move beyond the general rhetoric that often accompanies discussions of teacher change. The questions we asked in the survey and semi-structured interviews required them to provide specific mathematical examples to support their answers. In many of their responses, it was clear that not only had they learned new teaching strategies but new mathematics as well. Five themes emerged from the data collected in this study.

- Teachers lean to value informal strategies as mathematics
- Teachers see math differently- they make mathematical connections
- Teachers learn from their students
- Teachers describe a change in their classroom environment
- Teachers appreciated the tasks and the design of the curriculum.

Each theme will be discussed with direct quotes from teachers using examples from the MiC curriculum materials and professional development materials.

These themes will be elaborated throughout the rest of the article.

We present the results of this study from two perspectives. The first is our role as developers of the curriculum. The second perspective is that of staff developers. Over the past several years, each of us has been involved in helping teachers implement MiC through professional development workshops and classroom observations. With many of the participants in this study, we have developed a professional relationship that transcends the task of providing information about the curriculum. We have spent hours, through phone, email, and in-person conversation, sharing and reflecting on "best practices" with MiC. As a result, we, as researchers, have come to value their knowledge base and contributions to mathematics education

Background features of MiC and Professional Development Workshops

Imposing any new idea on teachers, such as new curricula or instructional strategy, is likely to be received with mixed results. MiC has been implemented by different school districts in a variety of formats ranging from full adoption across four grade levels to piloting of individual units at some grade levels. Like many of the new NSF-funded middle school curricula that are now available in commercial form, MiC was designed to meet the NCTM recommendations described in various landmark documents (1989, 1991, 1995).

Several unique features integral to MiC call for a comprehensive approach to professional staff development. One of these features is a developmental approach to mathematics instruction called *Realistic Mathematics Education* or

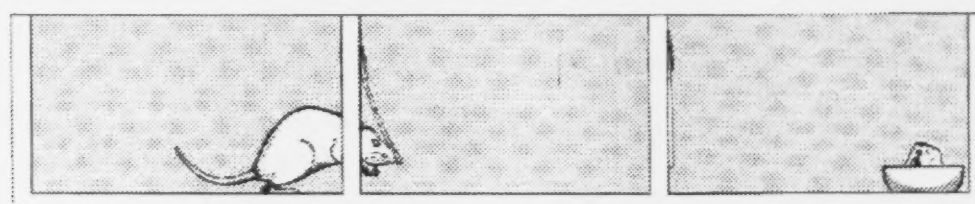
RME. The RME instructional approach focuses on guided reinvention of significant mathematics through the use of intermediary visual models that help students connect their knowledge of the context to formal mathematical content (Gravemeijer, 1994, 1995; Streefland, 1991; Treffers, 1991). Another feature is less emphasis on symbolic "skill and drill" than traditional math curricula and most reform-based curricula. In addition, most of the end-of-unit assessments are performance based, requiring students to apply their knowledge to new contexts and mathematical situations not previously encountered in the daily tasks. Each of these features became an important aspect of the professional staff development workshops that we conducted with these teachers and will be elaborated in the following sections.

Realistic Mathematics Education

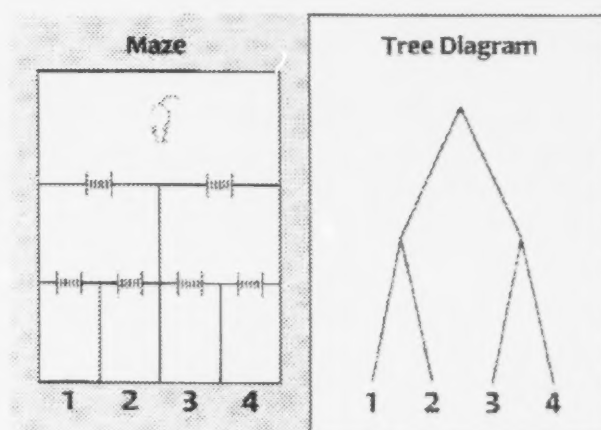
Traditional mathematics curricula typically present representations to illustrate a mathematical concept. This format assumes that students will gain insights into the concept by examining the visual. However we question this assumption about mathematical representations in light of new learning theories such as constructivism (Cobb, et al., 1992). In addition, the recently released document, *Principles and Standards for School Mathematics* (NCTM, 2000), has designated "representation" as one of ten Standards to consider across all of the grade levels. Furthermore, this document emphasizes representation as both "process and product....to the act of capturing a mathematical concept or relationship in some form and to the form itself"(p. 67). The design of MiC, incorporates a "bottom-up" approach to visual models so that students have an opportunity to connect their informal understanding of familiar contexts to

powerful mathematical concepts using formal mathematical representations (Gravemeijer, 1994, 1995).

These intermediary models are at the core of the MiC curriculum. For example, in Figure 1 taken from the first probability unit called *Take a Chance* (Jonker, et al, 1997), the visual model uses the context of a mouse traveling through a maze to provide students an opportunity to develop a tree diagram for listing all possible outcomes of a situation.



Each mouse has two different choices to make before reaching one of the final rooms. To describe the choices, you can use a picture called a **tree diagram**.



13. a. Use **Student Activity Sheet 4**. Put an H on the tree diagram at the place Harry would start.
- b. If Harry finds the food in room 3, trace Harry's path on the maze.
- c. Trace Harry's path on the tree diagram.

Robert wonders how the mice would behave in a different maze.

Figure 1 - MiC Unit *Take a Chance* page 32

Students go on to use this model to solve problems involving probability and expected value. The context provides a mnemonic tool that helps organize the

problem situation in a way that makes sense to the students. In MiC there are many powerful contexts, which incorporate organizing tools to promote students' understanding of important mathematical concepts.

Models such as this one are explored in depth during staff development workshops. We encourage teachers to examine these models from the students' perspectives and discuss how students might gain mathematical understanding. We have found that teacher's interactions with the models often provoke a variety of thought patterns and reflection. We find it critical in the workshops to help teachers learn about students' informal knowledge of key mathematical ideas. For example, in Figure 1, questions such as, "About how many [mice] would you expect to end up in room 3, where the food is?" and "Would exactly that many end up in room 3" were designed to elicit students' informal knowledge of expected value versus experimental trials (Jonker, et al., 1997, p. 32).

Number Models to Develop Computational Proficiency

The role of computation in MiC is perhaps the most controversial. With the increased role of technology in the form of calculators, the developers made a conscious decision **not to** include repetitious skill and drill in any part of the curriculum. This feature alone caused much concern for teachers implementing MiC, particularly during the first year. Thus, in workshops special attention is given to understanding how students utilize MiC number models to develop strategies leading towards computational proficiency.

The ratio table is one such number model used throughout MiC for all four operations involving whole numbers, fractions, decimals, and percents (see Figure 2). It is a useful tool in that it builds on students' knowledge of computing

with benchmark numbers (Brinker, 1998; Brendefur & Pitingoro, 1998; Streefland, 1991). This is one of the most popular models in the curriculum. Teachers find it fairly simple to implement and are encouraged by students' flexible ways of using the model to arrive at correct answers. Furthermore, the example in Figure 2 illustrates that there is an additive as well as multiplicative feature to this organizational recording device. The context is the driving force where by the tool or model provides an organizational structure that can be used across many different problem situations.

Mr. Martin's biology class is starting a school garden. They will be ordering plants by the box from a nursery. Mr. Martin asked the class to figure out how many tomato plants are in 16 boxes if one box contains 35 plants.

Three students—Darrell, Tasha, and Carla—solved the problem using ratio tables, but each student used a different table.

1. Darrell solved the problem as shown below:

Boxes	1	2	3	4	5	6	7	8	16
Plants	35	70	105	140	175	210	245	280	560

Explain Darrell's solution.

2. Tasha solved the problem as shown below:

Boxes	1	2	4	8	16
Plants	35	70	140	280	560

Explain Tasha's solution.

3. Carla solved the problem as shown below:

Boxes	1	10	2	6	16
Plants	35	350	70	210	560

Explain Carla's solution.

Figure 2 - MiC Number Tools Volume 1 page 36

During professional development workshops, we focus on teacher's informal mathematical knowledge and how students might "do number computation differently". As an example, during a MiC workshop we might ask teachers to solve the following discount problem:

The sale price on a coat was \$240 dollars after the original price was reduced by 20%. Bridget wants to buy the same coat. Unfortunately the sale is over. How much will Bridget have to pay for the coat?

Next we share the MiC number models with them. Finally we ask teachers to revisit the discount problem in at least two additional ways, using MiC number models. Just as students use MiC models flexibly, teachers learn the mechanics of the number models using a variety of strategies. This technique is particularly useful to help teachers improve their understanding of number and look at number from a fresh student perspective. Additionally, some MiC teachers find the ratio table and other number models to be useful tools in developing their own number sense. As a result of her use of this model, one of our participants exclaimed, "I think differently. For example, I make ratio tables in my head now!"

Assessment

Performance assessments are the norm rather than the exception in the MiC curriculum. For example, the assessment tasks for Take a Chance require students to not only answer questions about probability and chance, but also requires them to create sample spaces for non-standard math situations based on a given set of criteria. Figure 3 provides an illustration of one of these tasks.



Mr. Harris's and Ms. Lyne's classes were picked. Katrina is in Ms. Lyne's class. She wonders what to wear. She takes all of her blouses and slacks out of the closet to see what outfits she might wear. She finds that she has 12 different combinations of a blouse and a pair of slacks.

- b. Describe or draw the clothes Katrina might have taken out of her closet.

Figure 3 - MiC Unit *Take a Chance* Assessment, TG page 92

Frequently during professional development workshops, we present these types of tasks prior to a discussion of the individual MiC curricula unit in order to underscore the variety of ways that students might approach the problem. We use this technique as an informal assessment tool to assess teacher understanding of the mathematical content of the unit. This not only helps teachers to better understand the philosophy of the curriculum, but also helps them understand the overall goals of the particular unit. In addition, we continually focus on student thinking as a window to mathematical understanding. We encourage teachers to develop their own strategies to continually assess student understanding both formally and informally, using principles of a balanced assessment program.

To summarize, MiC professional development workshops focus on three key features. First, the examination of the intermediary models helps teachers gain an understanding of the underlying philosophy of the curriculum and helps them learn the mathematical content in the curricula materials from the students' perspective. Secondly, the underlying philosophy of computational proficiency across the curriculum is emphasized through discussion of the number models.

Finally, we focus on alternative forms of assessment, including observation and performance tasks.

Teacher Change

There is extensive literature on teacher change. Some pieces of research focus directly on teacher change, and others on new or different factors or programs, such as an experimental curriculum or a school wide structural reform, with teacher change as one of the outcomes. Much of the change literature conceptualizes the direction and process of change as that which is determined by someone other than the teachers who is going through the change process. Use of this concept of change often leads to the conclusion that change is difficult, and that teachers resist change. An alternative concept of change assumes that teachers change all the time (Richardson, et al., in press). These changes take place over the career of the teacher, and are largely voluntary.

Teachers have a considerable amount of discretion as to whether to implement the change in their classrooms. Research indicates that a teacher's knowledge can be altered through classroom interaction (Fennema & Franke, 1992). When knowledge is changed during instruction, that knowledge becomes more closely tied to the context in which it was developed. If the context in which the teacher is situated were to change because of different content, different classroom structure, or different students, the teacher's knowledge base would also change (Carpenter, Fennema, Peterson, & Carey, 1988; Shulman, 1987). The degree of change in the knowledge drawn upon depends on the source of the knowledge, how it is organized and connected, and the degree to which it is tied to a specific situation.

The findings in this study build upon two recent works related to teacher change and the MiC curriculum. One is a case study of two teachers implementing MiC. In this study, Clarke (1997) found that the [curriculum] unit of instruction and the quality of students' work on it had a major influence on one of the teachers who demonstrated observable change in his teaching approach. Specifically, after observing the various ways that students solved problems within the MiC units, this teacher regarded demonstration of strategies by himself as problematic and switched to more student-oriented lessons (Clarke, 1997, p. 300).

Another case study detailed the changing perceptions of one teacher who participated in the field-testing of initial versions of the MiC units (Meyer & Ludwig, 1999). This particular teacher reported that his "knowledge of mathematics and mathematics pedagogy" deepened as a result of teaching MiC (p. 266). Additionally, he found that, in contrast to his prior assessment techniques, he was now willing to go beyond evaluating students answers as simply right or wrong to potential use as "starting points for instruction"(p. 266).

Method

Fourteen middle school teachers (grades 5-8) and two support professionals across eight states (AZ, CA, IA, KA, NJ, NY, MI, & WI) volunteered to participate in the study. Each teacher had at least one year teaching MiC and had participated in one or more staff development workshops. Some of the participants evolved into consultants for implementing the program, participating

as staff development leaders to new MiC implementation sites. These teachers were selected because one or more of the authors noted a positive growth in their beliefs, math content and pedagogical content knowledge, and classroom assessment practices.

Survey instruments and semi-structured interviews were used as primary sources of data. The following are a selected sample of some of the survey questions used in this study.

- In what ways has your math instruction changed as a result of teaching MiC?
- Has your knowledge of mathematics changed as a result of teaching MiC?

In addition, participants were asked to participate electronically in reflections on their practice. At one point during the year, we asked teachers to predict how their students might solve a specific problem. Afterwards, we asked them to administer the problem to their students and reflect on their student's work. Anecdotal records from staff development workshops and classroom visits served as secondary data sources and were used to triangulate survey, reflection, and interview data (Glesne & Peshkin, 1992).

The survey was administered to each teacher individually and consisted of questions designed to provide baseline information about their epistemological beliefs, math content knowledge and pedagogical strategies for teaching middle grades topics in across the four the MiC content strands of Number, Algebra, Geometry & Probability and Statistics. Unlimited time was given to complete the survey and teachers were encouraged to be as explicit as possible in their answers to the questions.

Follow-up interviews were conducted with each of the participants either in person or over the phone. These interviews were semi-structured in nature and were used to clarify or add to responses given to the survey and/or to clarify reflection pieces. All fourteen teachers were surveyed and interviewed two times, prior to the beginning of the school year and before the end of the school year.

Self-Report Data

The use of teacher self-reports of their instructional practices poses several methodological challenges, such as the possibility of self-report bias due to social desirability, the potential ambiguity of questions, and that teachers' may have different perceptions of change than would a classroom observer (Spillane & Zeuli, 1999). In this study, we attempted to overcome this bias by asking teachers to reflect specifically on the contexts and content of the MiC units. We felt that, by pushing teachers to describe examples beyond general practice to those that focused on mathematical situations, some of the methodological limitations could be reduced.

These considerations in survey construction were based on several findings within the professional literature. One finding is that wording items that describe sensitive practices so that the questions do not require teachers to judge their own practices reduces response bias from social or professional concerns (Mullens & Gaylor, 1999). Porter et al. (1993) examined the correlation between teacher self-report survey data on detailed instructional practices and content coverage, teachers' classroom logs, and classroom observations. They

reported moderate to strong correlations between observation and log data, and between survey and log data.

Data Analysis

Comparative analysis was the primary method by which the data was organized and synthesized. The generative nature of our questions in the written survey instrument and the semi-structured interviews allowed us to synthesize the data by examining the patterns that emerged across the responses (Strauss & Corbin, 1994). Through this "inductive" strategy, we discovered a variety of themes that led to the grounding of our theoretical framework (Glesne & Peshkin, 1992). These themes will be detailed in the results section of the paper.

Results

Five themes emerged from the data collected in this study. Each theme will be followed by direct quotes from participants in the study. These quotes will be discussed with respect to both the underlying philosophy of the MiC curriculum and the current reform literature in mathematics education.

Teachers Learn to Value Informal Strategies as Mathematics

"I don't solve algebraic equations using the formal algebra. I have these convoluted strategies. Someone asked me at one company to do those challenging problems... I was able to solve the problem by using a drawing. I used to be afraid of not knowing the formal math...now I realize how much math power I have!"
(5th Grade Teacher)

"I have learned to value the low kids problem solving solutions"
(6th Grade Teacher)

"I would do a huge disservice to my kids by forcing kids to abstract too soon..."
(8th Grade Teacher)

"One thing that I have come to realize is how important it is to let students operate at less formal levels, rather than pushing to formal thinking and/or strategies. In the [unit] Graphing Equations, for example, it is very easy to give students the impression that the formal methods for solving equations are best, rather than drawing the model, if that is the method you illustrate on the board. I did this my first year, with the result that students who were not at that level tried to use the formal strategies and got confused and frustrated. I now model the picture model and the equations together every time I solve an equation or show the model if a student shows the equation. I have students using the method that makes sense to them." (8th Grade Teacher)

One of the most important features of the RME instructional approach is the emphasis on students' activities and informal ways of thinking about mathematics. The designers of the MiC curriculum carefully considered the development and sequencing of tasks to make the most of students' intuitive notions about the problems. The focus is more on problem solving with understanding than on perfecting the most formal procedures. The MiC philosophy encourages movement back and forth between the formal, pre-formal and informal methods. As referenced in the last quote above, this realization is often most challenging for those teachers that are well schooled in formal mathematics. The quote refers to a sequence of activities in the MiC unit *Graphing Equations* (Kindt, et al., 1998). Here, the process of solving a linear

equation with variables on both sides of the equal sign is presented through the context of jumping frogs as shown Figure 4.

Suppose that Alice and Fred travel the same distance with each jump, but Alice takes 5 jumps and Fred takes 3 jumps. The diagram below illustrates their new positions.



Figure 4 - MiC Unit *Graphing Equations*, page 32

Each frog begins a fixed distance from the path and makes jumps that are all the same length going in the same direction. Next students are presented with a problem (Figure 5) to solve. They are encouraged to share their strategy among other members in their class.

Suppose the frogs finish their jumps at exactly the same distance from the path, and you want to know the distance of each jump and each frog's final distance from the path. In groups, discuss strategies for solving this problem.

3. Share your group's method with the other members of your class.

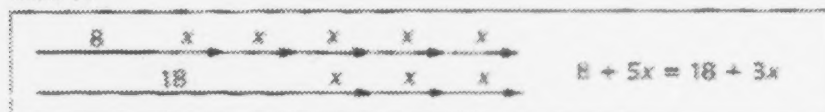
Figure 5 - MiC Unit *Graphing Equations*, page 32

Finally a visual model, accompanied by an equation, is presented as one of many solutions (Figure 6). Students are asked to explain the strategy depicted in each step. The situation is modeled using pictures that are closely tied to the frog context. Most importantly, these models provide access to informal

strategies that students and teachers may not otherwise consider if the equations would have been presented solely in symbolic form.

One way to answer problem 3 is to label the *unknown*. In this problem, the unknown is the length of each jump. You can use the symbol x for the length of a jump. The box below gives a diagram and an equation for answering problem 3.

Box A



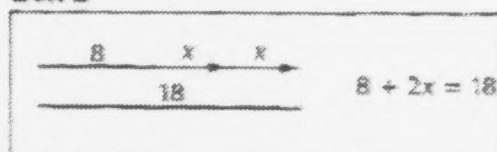
4. Explain how the equation $8 + 5x = 18 + 3x$ describes the diagram in Box A.



As steps in finding the length of a jump, look at the following diagrams and equations.

5. Explain the equation in Box B and describe how the diagram was changed from Box A to Box B.

Box B



6. Explain the equation in Box C and describe how the diagram was changed from Box B to Box C.

Box C

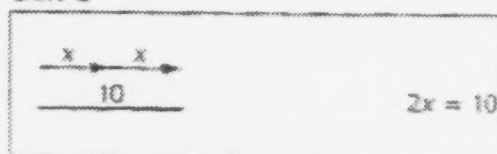


Figure 6 - MiC unit *Graphing Equations*, page 33

If students had difficulties, we recommended that teachers ask questions that would draw students back into the context to solve the problem. For example, in this situation we would encourage them to reflect back to the frog

context and draw the picture equations. In this way, as in many MiC units, the context becomes the driving force behind the mathematics, providing more students with access to more mathematics. Contrast this approach with a traditional algebra course where only the symbolic form is presented to students and formal procedures are shown for students to emulate and copy. The process of solving linear equations thus becomes a practice of manipulating numbers and symbols.

The data of this study supports the claim that valuing informal solution strategies not only provides more students with mathematical power, but also provides teachers a different approach and opportunity to learn mathematics. The 8th grade teacher making reference to the Frog situation provides further evidence of how her perception of mathematics changed as a result of teaching MiC. She now values a variety of strategies, both informal and formal, as legitimate mathematical solutions to algebraic problems.

Teachers See Math Differently -They Make Mathematical Connections

"Ratio tables helped me...I visualize it for mental math...I use percent bars...like if I am at the store.. I finally learned number sense..."

(5th Grade Teacher)

"Students make connections but not always in the way you expect."

(5th Grade Teacher)

"I finally see that mathematics can be more than just arithmetic."

(7th Grade Teacher)

"Some of the connections have been good. Like slope and contour maps."

(7th Grade Teacher)

"I think it is most important for students to understand what they are learning, to develop a strong conceptual understanding so that procedures make sense."

(7th Grade Teacher)

"I never had the scatter plots and other data representations...in the graphing units too. It is cool. I like it. The models, ratio table, bar models...they help

kids in a very concrete way.... Like in Expressions & Formulas those 2 or 3 variable problems it's a new approach to set up things. It is really different and powerful"
(6th Grade Teacher)

"I've learned more about the connections in mathematics between different topics that we typically promote as being so distinct. For instance, in one of the 8th grade geometry units, the following topics are all grouped together: Pythagorean theorem, glide ratios, perpendicular bisectors, areas of different figures, circumference of a circle, polyhedral shapes. Then the most amazing thing happens. The assessment at the end of the unit actually ties it all together in a problem set up to assess all these things."

(8th Grade Teacher)

The last quote above refers to end-of-unit assessment task for the MiC unit *Going the Distance* (Abels, et al., 1998). The problem, shown in Figures 7 & 8, requires students to apply the concepts they have learned to a new situation.

There is a shortage of parking places in downtown Detroit, Michigan. The city council decided to convert the rooftop of a high-rise building into a parking garage. In order to accomplish this, an architect was hired to design a model for a ramp that cars can use to reach the rooftop parking area. The city codes mandate that the grade of the road cannot exceed 10%, or in other words, the slope must be smaller than 0.1 or $\frac{1}{10}$. The building is 24-meters high.

1. a. On a piece of graph paper, make a scale drawing of the straight ramp with a slope of 0.1. (Use a scale of 1 centimeter = 6 meters.)
- b. How long is the ramp?

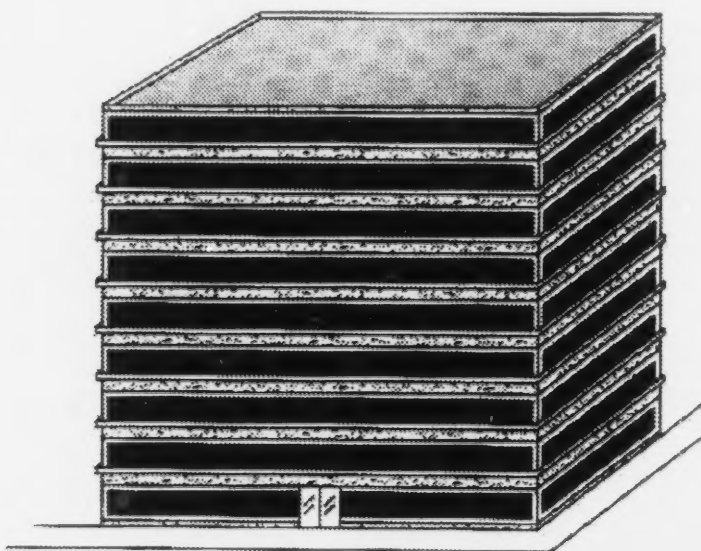
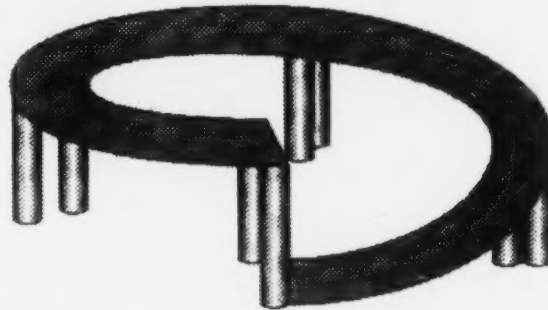


Figure 7 - MiC unit *Going the Distance* Assessment, TG page 139.

The architect sees that a straight ramp will extend too much into the street. Therefore, she decides to turn her straight ramp into a circle that will be attached to the side of the building. The ramp is 5 meters wide.



2. a. What will be the diameter of the circular open area if the ramp has one complete turn?
- b. If the ramp is redesigned so that it maintains its length but makes two complete turns, what will be the diameter of the circular open area?

The architect decides to build the circular ramp with two complete turns. The ramp will be five meters wide. The circular open area inside the ramp will be used to erect an elevator for people to travel from the rooftop parking area to the street level.

3. What will be the area of the elevator floor if the elevator uses all the available space?
4. Is there a difference between the slope of the inner edge and the slope of the outer edge of a circular ramp? If so, describe the difference.

Figure 8 - MIC unit *Going the Distance* Assessment, page 140

The teachers of this study shared that they now see math differently. Some of them were math phobic, others were taught math as fragmented topics in isolation. It appears that all of them in some way or another now see more math connections. They understand math differently. They claim that the curriculum helps them and their students to see more of the big picture about mathematics.

Teachers Learn from Their Students

"I've learned that children's thinking can go much deeper and beyond what I had previously been expecting of them"

(6th Grade Teacher)

"Issues of multiple strategies... I think that there are so many different ways to solve a problem... Whether we got it right or not...the fact that I never thought so much about it before...It is so funny, now that's all we think about in class. Each of the kids thinks so differently..."

(5th Grade Teacher)

"I just learned another connection from a kid recently. I am an algebraist. I am not very good at geometry. We were tessellating triangles in Triangles & Patchwork. One of my students solved the problem in a very visual way. I am learning to appreciate the visual approach more and more"

(8th Grade Teacher)

"I'm more willing to see the mathematics the way kids see it."

(7th Grade Teacher)

"Most new mathematics they [the teachers] learn is through teaching the materials. We all know that the teacher learns more than anyone. However, there are specific instances when I have observed teachers learning through student's eyes; e.g. a student presents a strategy, and the teacher has to say, 'wait a minute and let me think about that.'"

(Math Support Person)

"In working on this geometry unit, I overlooked the idea that we were drawing a perpendicular bisector between two points. We had been drawing perpendicular bisectors using a compass and a straight edge. Since I wasn't thinking of it by that name, I overlooked the fact that you could simply connect two points, find the midpoint using a ruler, make a 90 degree angle there, then draw the line (i.e. the perpendicular bisector). However, one of my students noticed that as a method and presented it to the class. At this point, I recognized what I'd been missing, and knowing this, asked the students what we should call this type of line that we'd been drawing in. One of the students noted that we'd drawn it perpendicular to that "other" line and another said we cut it in half. It wasn't hard to convince them that "perpendicular bisector" fit the bill."

(8th Grade Teacher)

The *Standard* that often receives a great deal of attention is

"Communication". The consensus is that the ability to articulate mathematical ideas is important for student learning. It has also been found to be an important instructional tool for teachers. That is, students' thinking can provide the basis for teacher's decision-making (Carpenter, et al., 1988). However, this

study has also shown that the design and implementation of the MiC curriculum can also provide teachers with opportunities to learn new mathematics by assessing their students' approaches to solving problems in the MiC curriculum. This is particularly true for teachers who may not have as much formal mathematical background or a limited understanding of mathematics.

Because traditional texts and methods usually present the most "efficient mathematical strategy", teachers often times do not know themselves that other strategies are legitimate. By focusing on the students' strategies for solving problems, MiC teachers often find that they learn new mathematics as well.

Teachers Describe a Change in their Classroom Environment

"Instead of standing up and showing them how to do everything. I provide a place and mind set to solve problems... Before I use to give them strategies to solve problems, now they give me the strategies... I assess kids differently."

(5th Grade Teacher)

"I can facilitate a discussion better...I ask better questions...I still need work but I am getting better! I focus more on process and concepts than before."

(5th Grade Teacher)

"The biggest change I see is that teachers (and students) are once again excited about mathematics."

(Math Support Person)

"My kids don't want to stop when time is up."

(5th Grade Teacher)

"I could see kids being more engaged in mathematics; really solving problems. Recently we had our Formal District Assessments. It used to be that kids would just fill in the bubbles on the answer sheet and not even try to do the problems. This time, all the kids were attacking the problems and really getting into it. They've gained confidence...they have an arsenal of strategies... before they would have sat and done nothing...now they have plan of attack. They are risk takers"

(8th Grade Teacher)

Learning new mathematical strategies from their students has transformed many MiC teachers' classrooms. In response to the question

"Has your teaching of mathematics been influenced since implementing MiC? Describe how it has changed", the majority of the participants described a classroom that is student centered. This comment was consistent across the responses and reflected a change in perception about their role as a math teacher.

While each individual teacher's classroom is unique, during observations in many of the participating teachers' classrooms, we have noticed several similarities in the norms and practices that have been established as a result of implementing MiC. Most of these teachers use cooperative groups as the norm rather than the exception or special occasion. Most also focus class discussions on mathematical discourse and sharing of students' strategies for solving the problems. This is exemplified by one of the participants who stated, "Students are a bigger part of the instruction - they teach too". Finally, these teachers have placed more emphasis on observational data in assessing student progress and discovered creative ways to document what they observed.

Teachers Appreciate the Tasks and the Design of the Curriculum

"For example in Graphing Equations problem 19 where the kids are given two lines that will intersect off the graph. The strategies the kids come up with to find the point of intersection is amazing! Kids come up and share how they did it. It is very powerful. I'd like to do it more...Problems like that should be in neon signs. They are great! You can see development from informal to formal"

(8th Grade Teacher)

"The curriculum has given me more clarity into what student's can do. I know what to expect from and watch it and can nurture it"

(6th Grade Teacher)

"I like the MiC approach because it gives students a chance to think it through on their own first."

(5th Grade Teacher)

"I no longer aim only for mastery of each topic before moving on. ...I've seen how concepts continually reappear and how this spiral provides opportunities for students to engage in concepts again and again and so build knowledge on a broader foundation"
(6th Grade Teacher)

"I can remember when I taught Decision Making... I came to our next workshop and showed you the amazing student work. I would have never believed in a million years that these lower level kids could use such informal strategies to graph lines. The context provided my students with access to some pretty sophisticated mathematics. Then it happened again when you shared the bar model with us. I tried it out right away with my 8th grade students. More kids were finally able to master those percent problems. The percent bar helped them make sense of the problems. You know I even forgot that was new. I have just been using for so long now it seems second nature."
(8th Grade Teacher)

We experienced many challenges and some frustrations as we worked with others to make MiC meaningful for students and teachers. In working on individual units, many American developers did not appreciate the uniqueness of MiC. However, just as teachers come to appreciate mathematical activities when they witness their own students' successes with them, we have come to appreciate the benefits of the curriculum as teachers describe their successes with MiC. In particular, they are able to point to specific tools that have helped them become better mathematical thinkers, such as the case with the ratio table or bar model. It convinces us that teachers can learn content that they did not previously understand by following the developmental process that their students follow as they work through the problems in the units.

Conclusions

Attention now turns to an examination of the revised curriculum *Standards* (2000) document. The five themes described in this article reflect success stories of the original document and anticipate new goals for mathematics

education as envisioned in the new document. The six principles that guide the *Standards* (2000) document are equity, curriculum, teaching, learning, assessment, and technology. The Teaching Principle is stated as follows, "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p. 16). We feel that the MiC curriculum has provided teachers with an innovative way to view students' informal knowledge of mathematics through the visual and dynamic representations used to introduce and develop the concepts.

The Curriculum Principle goes onto say, "A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades" (p. 14). From a developer and staff developer's perspective, we appreciate the connections across the MiC curriculum. We also recognize that individual teachers at any particular grade level may not have developed this level of understanding. We attempt, as much as possible, to provide teachers with opportunities to experience these connections during the staff development workshops. In some cases, teachers like the one quoted at the beginning of this article, completely change their perception of the entire subject of mathematics as a result of teaching the MiC curriculum. Many of them reflect on the distinctions between their current understanding of mathematics and their knowledge prior to their implementation of the curriculum.

One of the supervisors who participated in the study shared her perceptions of how the implementation of MiC has impacted the teachers in her district. She noted in the following quote that MiC provided benefits to teachers

whom already have a good knowledge base of mathematics and those who may not.

For those who know the math, it has radically changed their mode of delivery. For those teachers who do not know the math, it provides them a way to teach math in a non-threatening way.

(MiC District Support Person)

Many of the teachers in this study are not math certified. Teaching the MiC curriculum has helped them learn new mathematics as their students experience the activities and problems in the units.

The underlying philosophy of the MiC curriculum emphasizes students' developmental learning patterns and assessment. The teachers' perceptions of how their own instruction has changed as a result of teaching MiC provides much potential for the envisioned goal that all students learn significant math topics. However, the equity principle remains as the most challenging to incorporate. Emphasis on valuing multiple strategies provides students who do not typically perform well on traditional math tasks with opportunities to learn concepts that in the past were gatekeepers to more advanced math classes in high school. Still, we face ongoing challenges in how to help teachers help all students learn significant mathematics as envisioned by this guiding principle. Much too often we continue to see the need for district to sort students; this is even after they see the beauty of allowing kids to successfully solve the same problems at different levels. More is needed in the way of longitudinal studies of students' achievement across grade levels. Currently, some are underway. While we cannot say that MiC has worked for all teachers in all districts, the quote we end with provides a taste of the potential impact that this and other Standards based

curricula could have on the performance of all students in middle grades mathematics.

I could see kids being more engaged in mathematics; really solving problems. Recently we had our Formal District Assessments. It used to be that kids would just fill in the bubbles on the answer sheet and not even try to do the problems. This time, all the kids were attacking the problems and really getting into it. They've gained confidence...they have an arsenal of strategies... before they would have sat and done nothing...now they have plan of attack. They are risk takers.

(8th Grade Teacher)

References

- Abels, M., de Lange, Jan, Brendefur, J. Halevi, T. (1998). Going the distance. In National Center for Research in Mathematical Sciences Education & Freudenthal Institute (1997-98), (Eds.), *Mathematics in context: A connected curriculum for grades 5-8*. Chicago: Encyclopaedia Britannica Educational Corporation.
- Ball, D. (1991). Research on teaching mathematics: Making subject matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching* (pp1-48). Greenwich, CT: JAI Press.
- Brinker, L. (1998). Using recipes and ratio tables to build on students' understanding of fractions. *Teaching Children Mathematics*, 5 (4), 218-224.
- Carpenter, T., Fennema, E., Peterson, P.L., & Carey, D. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, 19(5), 385-4.
- Civil, M. (1995). Listening to students' ideas: Teacher interviewing in mathematics. In L. Meira & D Caraher (Eds.), *Proceedings of the 19th Psychology of Mathematics Education conference* [Volume 2] (pp. 2:154-2:16). Brazil: Universidade Federal do Pernambuco.
- Clarke, D.M. (1997). The changing role of the mathematics teacher. (1997). *Journal for Research in Mathematics Education*, 28(3), 278-308.
- Cuban, L. (1988). Constancy and change in schools (1880's to the present). In P. Jackson (Ed.), *Contribution to educational change: Perspectives on research and practice* (pp. 85-106). Berkeley, CA: McCutcheon.
- Fennema, E., Carpenter, T., & Peterson, P. (1989).....
- Fennema, E., & Franke, M.L. (1992). Teacher's knowledge and its impact. In D.A.Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.
- Glesne & Peshkin (1992)
- Hall, G.E., & Loucks, S. (1977). A developmental model for determining whether the treatment is actually implemented. *American Educational Research Journal*, 14(3), 263-276.

- Hiebert, J. (Ed.). (1986). *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Erlbaum.
- Jonker, ... (1997). Take a chance. In National Center for Research in Mathematical Sciences Education & Freudenthal Institute (1997-98), (Eds.), *Mathematics in context: A connected curriculum for grades 5-8*. Chicago: Encyclopaedia Britannica Educational Corporation.
- Kindt, M., Wijers, M., Spence M., Brinker L., Pligge M. Burrill J. (1998). Graphing Equations. In National Center for Research in Mathematical Sciences Education & Freudenthal Institute (1997-98), (Eds.), *Mathematics in context: A connected curriculum for grades 5-8*. Chicago: Encyclopaedia Britannica Educational Corporation.
- Lappan, G. (1997). The challenges of implementation: Supporting teachers. *American Journal of Education*, 106. The University of Chicago, pp. 207-239.
- Meyer, M., & Ludwig, M. (1999). (title), *Mathematics Teaching in the Middle School*
- Mullens, J. E., & Gaylor, K. (1999). Measuring classroom instructional processes: Using surveys and case study field test results to improve item construction. U.S. Department of Education, Office of Educational Research and Improvement. Washington, DC.
- National Center for Research in Mathematical Sciences Education & Freudenthal Institute (1997-98), (Eds.), *Mathematics in context: A connected curriculum for grades 5-8*. Chicago: Encyclopaedia Britannica Educational Corporation.
- National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics (1991). *Principles and Standards for Mathematics Teaching*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics (1995). *Assessment Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

- Resnick, L., & Ford, W. (1981). *The psychology of mathematics for instruction*. Hillsdale, NJ: Erlbaum.
- Richardson, V., & Placier, P. (in press). Teacher change. In V. Richardson (Ed.), *Handbook of research on teaching*. (4th edition). Washington, DC.: American Educational Research Association.
- Romberg, T.A. (1992). Problematic features of the school mathematics curriculum. In Philip Jackson (Ed.), *Handbook of research on curriculum* (pp. 749-788). New York: Macmillan
- Schifter, D. (1995). Teachers' changing conceptions of the nature of mathematics: Enactment in the classroom. In B.S. Nelson (Ed.), *Inquiry and the development of teaching: Issues in the transformation of mathematics teaching*. Center for the Development of Teaching Paper Series. Newton, MA: Center for Development of Teaching, Education Development Center, Inc.
- Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1-22.
- Spence, M. 1996: Psychologising Algebra: case studies of knowing in the moment, unpublished PhD thesis, Madison Wisconsin.
- Spillane, J. P., & Zeuli, J. S. (1999). Reform and teaching: Exploring patterns of practice in the context of national and state mathematics reforms. *Educational Evaluation and Policy Analysis*, Spring, 1999, 21(1), 185-203.
- Strauss, A., & Corbin, J. (1994). Grounded theory methodology: An overview. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research*. Thousand Oaks, CA: SAGE Publications.
- Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal*, 28(3), 587-616.